XI EMED - Encontro Mineiro de Equações Diferenciais
20 a 22 de Novembro de 2017
Poços de Caldas-MG

Stochastic continuity equation with non-smooth velocity

David Alexander Chipana Mollinedo
Universidade Tecnológica Federal do Paraná - UTFPR - PR
Av Monteiro Lobato, s/n - Km 04
84016-210, Ponta Grossa, PR
E-mail: davida@utfpr.edu.br

In this work we study the following stochastic linear transport/continuity equation

$$\begin{cases} \partial_t u(t,x) + Div\big((b(t,x) + \frac{dB_t}{dt}) \cdot u(t,x)\big) = 0\,, \\ u|_{t=0} = u_0\,, \end{cases} \tag{1}$$

where $(t,x) \in [0,T] \times \mathbb{R}^d$, $\omega \in \Omega$ is an element of the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, $b : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d$ is a given vector field and $B_t = (B_t^1, ..., B_t^d)$ is a standard Brownian motion in $\mathbb{R}^d$. The stochastic integration is to be understood in the Stratonovich sense. A very interesting situation is when the stochastic problem is better behaved than the deterministic one. A first result in this direction was given by F. Flandoli, M. Gubinelli and E. Priola in [1], where they obtained well-posedness of the stochastic problem (1) for an Hölder continuous drift term, with some integrability conditions on the divergence. Their approach is based on a careful analysis of the characteristics.

Só, the main issue of this work is to prove uniqueness of L^2-weak solutions for one-dimensional stochastic continuity equation (1) with unbounded measurable drift without assumptions on the divergence. More precisely, we assume that b satisfies

$$|b(x)| \leq k(1 + |x|).$$

The proof is based in the fact that one primitive V is regular and it verifies the transport equation

$$\partial_t V(t,x) + (b(t,x) + \frac{dB_t}{dt}) \cdot \nabla V(t,x) = 0\,. \tag{2}$$

Then using a modified version of the "commutator Lemma" and the characteristic systems associated to the stochastic partial differential equation (2) we shall show that $V = 0$ with initial condition equal to zero , which implies that $u = 0$. For more details of this result to see [2]. Joint work with Christian Olivera (Universidade Estadual de Campinas).

Referências

[1] F. Flandoli, M. Gubinelli and E. Priola, *Well-posedness of the transport equation by stochastic perturbation*, Invent. Math., 180, 1-53 pages, 2010.

[2] D. A. C. Mollinedo and C. Olivera, *Stochastic continuity equation with non-smooth velocity*, Annali di Matematica Pura ed Applicata (1923 -), page 1-16, 2017.

Palavras-chave: *Stochastic partial differential equation, Continuity Equation, Stochastic characteristic method, Regularization by noise, Commutator Lemma.*