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Remarks about a fractional Choquard equation: ground state, regularity and polynomial decay

Pedro Belchior, **Hamilton Bueno**, **Gilberto Assis Pereira**,
Departamento de Matemática - UFMG
E-mail: pedrobelchior18@gmail.com, hamilton@mat.ufmg.br, gilbertoapereira@yahoo.com.br
Olímpio Miyagaki
Departamento de Matemática - UFJF
E-mail: ohmiyagaki@gmail.com

We consider a Choquard-type equation in a fractional (s,p)-Laplacian setting:

$$(-\Delta_p)^s u + A|u|^{p-2}u \;=\; \left(\frac{1}{|x|^\mu} * F(u)\right) f(u) \quad \text{in } \mathbb{R}^N, \tag{1}$$

where A is a positive constant, $0<\mu<N$, $F(t)=\int_0^t f(r)\mathrm{d}r$ and $(-\Delta_p)^s$ denotes the (s,p)-Laplacian operator.

In (1), we assume that f is a C^1 function that satisfies

(f1) $\displaystyle\lim_{t\to 0}\frac{|f(t)|}{t^{p-1}}=0$;

(f2) $\displaystyle\lim_{t\to\infty}\frac{f(t)}{t^{q-1}}=0$ for some $p<q<\frac{p_s^*}{2}\left(2-\frac{\mu}{N}\right)$, where

$$p_s^*=\frac{Np}{N-sp} \quad\text{with}\quad sp<N;$$

(f3) $f'(t)t^2-(p-1)f(t)t>0$ for all $t>0$.

Since we are looking for positive solutions, *we suppose that* $f(t)=0$ *for* $t\le 0$.

We summarize our results:

Theorem 1. *Suppose that, for* $p<q<(N-\mu)p/(N-p)$, $0<\mu<N$, *conditions* (f1)-(f3) *are valid. Then, for any* $A>0$, *problem* (1) *has a non-negative ground state solution* v.

The first result was shown by proving that the natural energy functional attached to this problem satisfy the mountain pass geometry. By introducing the Nehari manifold, we show that the minimizing sequence converges strongly.

Supposing that $0 < \mu < p$, we adapt ideas from Brasco, Mosconi e Squassina [2, Proposition 3.2] and prove additionally that the ground state solution is continuous:

Theorem 2. *Let $v \in W^{s,p}(\mathbb{R}^N)$ be a non-negative solution of problem* (1)*. Then $v \in L^\infty(\mathbb{R}^N) \cap C^0(\mathbb{R}^N)$.*

Finally, we prove that the ground state solution has polynomial decay:

Theorem 3. *Suppose that* (f1)-(f3) *hold and $0 < \mu < p < q < \frac{p^*}{2}\left(2 - \frac{\mu}{N}\right)$. Then, there exist constants $\rho > 0$ and $C > 0$ such that the solution obtained in Theorem 1 satisfies*

$$v(x) \leq \frac{C}{|x|^{\frac{N-sp}{p-1}}} \quad \forall\, x > \rho.$$

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