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On the problem of ternary mixture of elastic materials

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Abstract

We consider a mathematical model that describes a mixture of three elastic materials. We study, from the functional and numerical analysis point of view, the mixture problem with frictional damping. The problem consists of a linear system of three coupled hyperbolic partial differential equations. An existence and uniqueness result and an energy decay property are mentioned.

we consider the one dimensional model of interacting continua over the compact interval $[0, l]$.

$$\mathbf{R}U_{tt} - \mathbf{A}U_{xx} + \mathbf{N}U + \mathbf{B}(x)U_t = 0, \tag{1}$$

$$U(0,t) = U(l,t) = 0, \quad t \in \mathbb{R}^+, \tag{2}$$

$$U(x,0) = U_0(x), \quad U_t(x,0) = U_1(x). \tag{3}$$

with $U = (u^1, u^2, ..., u^n)$ and

$$\mathbf{R} = (\rho_i \delta_{ij})_{n\times n}, \quad \mathbf{A} = (a_{ij})_{n\times n}, \quad \mathbf{B} = (b_{ij}(x))_{n\times n}, \quad \mathbf{N} = (n_{ij})_{n\times n}$$

where δ_{ij} is the Kronecker's delta, $\mathbf{A}$ is a positive definite (real) symmetric matrix and $\mathbf{B}$ and $\mathbf{N}$ are semipositive definite (real) symmetric matrices. In [2] Dell'Oro and Rivera, made a full characterization of the asymptotic behavior of the (1)-(3) when $n = 3$, $N = \left((-1)^{i+j}\right)$ and $\mathbf{B}$ is a constant diagonal matrix. In [1] F. Córdova and Rivera, show the lack of polynomial stability to the corresponding semigroup of (1)-(3) when $\mathbf{N} = 0$ and $\mathbf{B}$ is a constant matrix, here we consider that if $B(x) = C^T D(x) C$ and $D(x)$ is a full rank and $\mathbf{N} = C^T D_0 C$ then the corresponding semigroup is exponentially stable if and only if the imaginary axis is contained in the resolvent set of the infinitesimal generator. In particular this implies the lack of polynomial stability to the corresponding semigroup. We performed numerical experiments in order to exemplify the concepts of exponential stability obtained.

Referências

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Palavras-chave: *Exponential Stability, Semigroups, Mixture of Materials, Finite Difference Method, Numerical Simulations.*