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A 3D elliptic PDE computational model for Darcy flow linked to practical petroleum reservoir engeneering

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In this work, we study a numerical approximation procedure for solving a fundamental differential problem linked to practical petroleum reservoir engeneering [3], namely, the three-dimensional quarter of five spot related to a single phase Darcy flow in porous media, modeled by a scalar second order partial differential equation of elliptic nature [4], in a compact domain $\Omega \subset \mathbb{R}^3$ with Lipschitz boundary. Indeed, the mathematical model at hand (e.g., [1, 2]) relates the Darcy's velocity $\mathbf{u} \in H(div, \Omega)$ with the pressure $p \in L^2(\Omega)$, through the system

$$\nabla \cdot \mathbf{u} = f(x), \qquad \mathbf{u} = -\mathbf{K}(x)\nabla p, \quad x \in \Omega \subset \mathbb{R}^3, \tag{1}$$

along with the Neumann boundary condition

$$u \cdot \nu = 0, \quad x \in \partial\Omega, \tag{2}$$

and the zero mean pressure constraint

$$\int_\Omega p \, d\Omega = 0, \tag{3}$$

where $\mathbf{K}(x) \in \mathbb{R}^{3\times3}$ is the symmetric positive definite (SPD) absolute permeability full tensor [3], which may be heterogeneous, ν is a unity vector normal to $\partial\Omega$, and f is the source term that models injection well ($f(x) = 10^{-7}$ if $x = (0,0,0)$), production well ($f(x) = -10^{-7}$ if $x = (1,1,1)$), and 0 in all other cases; see Figure 1. It is worth mentioning that the single scalar constraint on the pressure (3) is needed because (1)-(2) determine the pressure only up to an additive constant. The zero mean constraint (1) is one way to fix that constant (by setting the pressure value at a point is another way to remove the nullspace, but fairly inappropriate for practical purpose in real industrial applications).

The impact of the particular numerical method employed for solving the fundamental differential Darcy problem (1)-(3) in a broad range of large scale industrial applications in high-contrast discontinuous heterogeneous porous media $K(x)$ (in the absence of analytical methods), must be twofold [1, 2]: firstly, the approximation space to the pair (p, u) must be chosen such that the Darcy flux u be *conservative* and secondly, the discretization of the second order elliptic operator should be consistent to the underlying coercive self-adjoint elliptic operator, yielding in a well-conditioned SPD linear system to be solved efficiently (possibly with an iterative procedure). To this end, we used a conservative hybrid mixed finite element method (HMFEM) using lowest order Raviart-Thomas-Nedelec ($\boldsymbol{RT_0}$) elements for the approximation of the pressure-velocity Darcy system [1, 3] to discretize (1)-(3), along with a domain decomposition technique. The HMFEM simultaneously approximates $\boldsymbol{u}$ and $\boldsymbol{p}$ and guarantees local (and global) conservation of $\boldsymbol{u}$ [1, 2]. The finite element velocity space consists of piecewise linear functions with continuous normal components on the cubes interface [1, 2], while the pressure space consists of piecewise constant functions.

To the practical petroleum reservoir engeneering at hand (1)-(3), we choose to construct a global linear system for the whole (discretized) domain $\boldsymbol{\Omega}$ and solve the pertinent linear algebra problem by using a C++ academic software, namely, the Algebraic Multigrid (AMG), which offers a precise and fast solving of SPD linear systems.

We present numerical approximations for the differential model at hand (1)-(3) related to the $\boldsymbol{1/4}$ five-spot 3D problem aiming flow diagnostics in reservoir management for improving oil recovery (see Figure 1 for the Darcy velocity field to this model). We conclude that the quality of the numerical solution is satisfactory and resembles main features of the Darcy flow as expected for this real industrial benchmark problem.

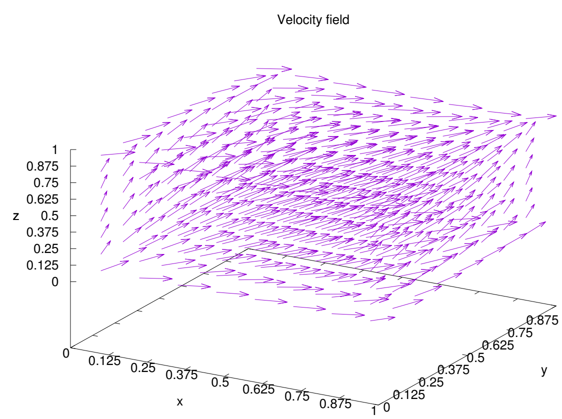

Figure 1: Velocity field profile approximated by piecewise linear functions with continuous normal components on the cubes interface, considering a $\boldsymbol{8 \times 8 \times 8}$ grid in the $\boldsymbol{\frac{1}{4}}$ five spot practical petroleum reservoir engeneering problem.

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References

[1] J. Douglas, P. J. Paes Leme, J. E. Roberts, and J. Wang. A parallel iterative procedure applicable to the approximate solution of second order partial differential equations by mixed finite element methods. *Numerische Mathematik*, **65**(1) (1993) 95–108.

[2] A. Francisco, V. Ginting, F. Pereira, and J. Rigelo. Design and implementation of a multiscale mixed method based on a nonoverlapping domain decomposition procedure. *Mathematics and Computers in Simulation*, **83**(3) (2014) 353–369.

[3] M. G. Gerritsen and L. J Durlofsky. Modeling fluid flow in oil reservoirs. *Annu. Rev. Fluid Mech.*, **37** (2005) 211–238.

[4] R. J. Iorio Jr and V. M. Iorio. *Fourier analysis and partial differential equations*, **70**. Cambridge University Press (2001).